# Probability and Random Processes

## ECS 315

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## 8 Discrete Random Variable

**Office Hours:**

BKD, 6th floor of Sirindhralai building

| | |
|---|---|
| Tuesday | 9:00-10:00 |
| Wednesday | 14:20-15:20 |
| Thursday | 9:00-10:00 |

# Discrete Random Variable

- $X$ is a **discrete** random variable if it has a countable support.
  - Recall that countable sets include finites set and countably infinite sets.
- For $X$ whose support is uncountable, there are two types:
  - **Continuous** random variable
  - **Mixed** random variable

# Example: pdf and probabilities

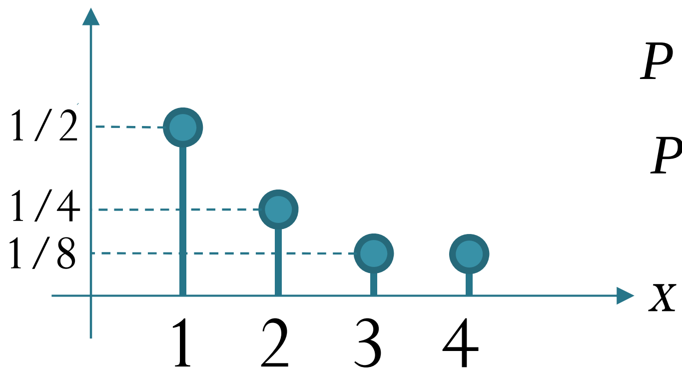
Consider a random variable (RV) $X$.

probability mass function (pmf)

$$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

$$p_X(x) = \mathrm{P}[X = x]$$

stem plot:




$$P[X = 2] = ?$$

$$P[X > 1] = ?$$

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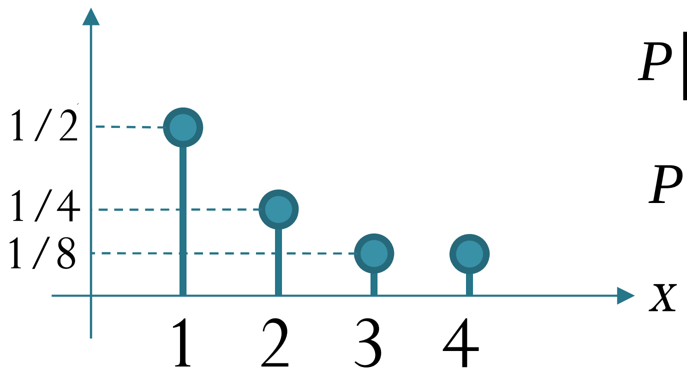


$$P[X = 2] = p_X(2) = \frac{1}{4}$$

$$P[X > 1] = p_X(2) + p_X(3) + p_X(4)$$
$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$$

# Example: pdf and its interpretation

Consider a random variable (RV) $X$.

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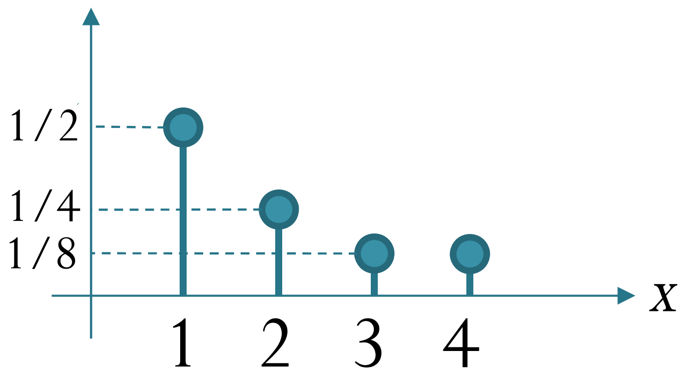

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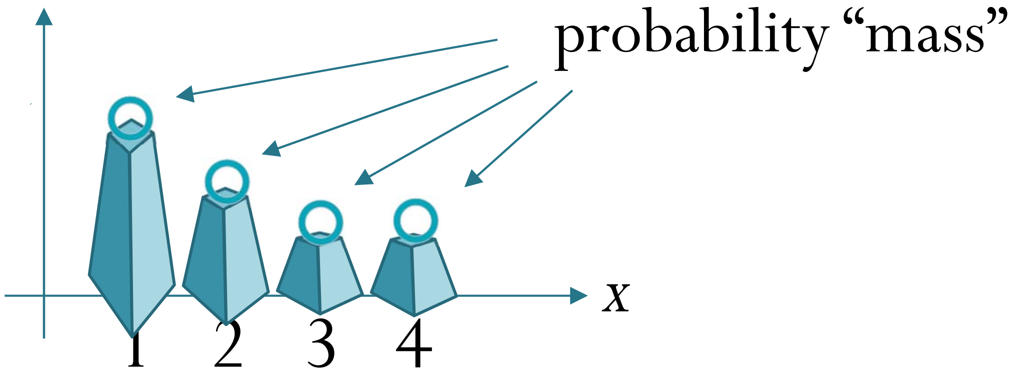

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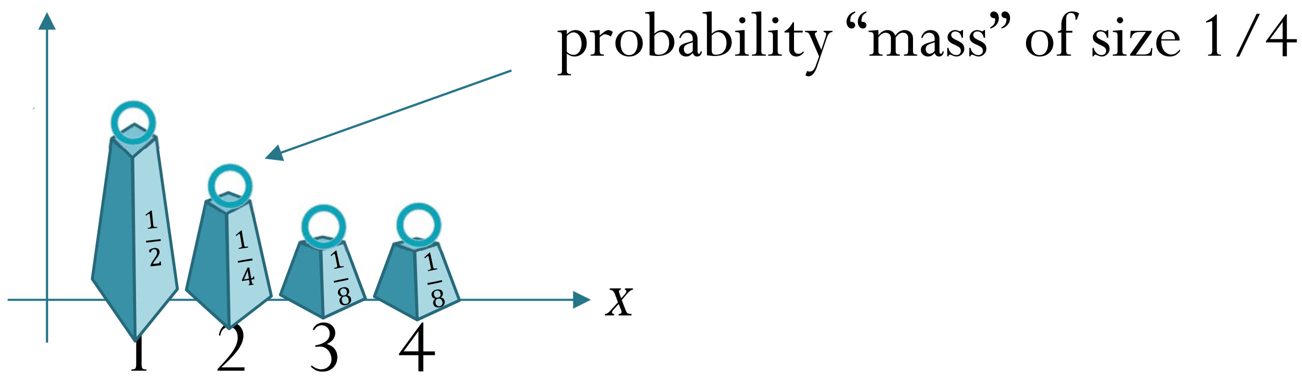

# Example: Support of a RV

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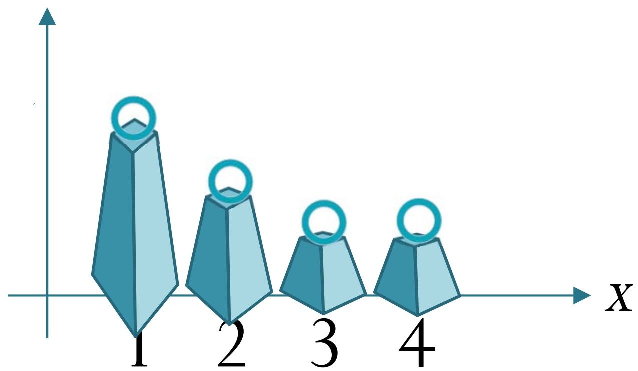


What about the support of this RV $X$?

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The set $\{1,2,3,4\}$ is a support of $X$.

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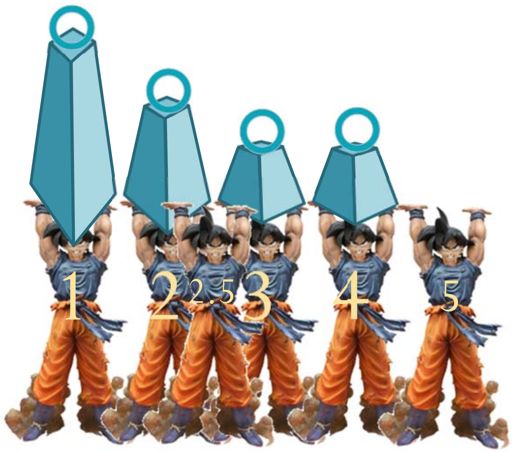


The set $\{1,2,2.5,3,4,5\}$ is also a support of this RV $X$.

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The set $\{1,2,4\}$ is *not* a support of this RV $X$.

# Example: Support of a RV

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The set $\{1,2,3,4\}$ is the "minimal" support of $X$.

For discrete RV, we take the collection of $x$ values at which $p_X(x) > 0$ to be our **"default" support**.

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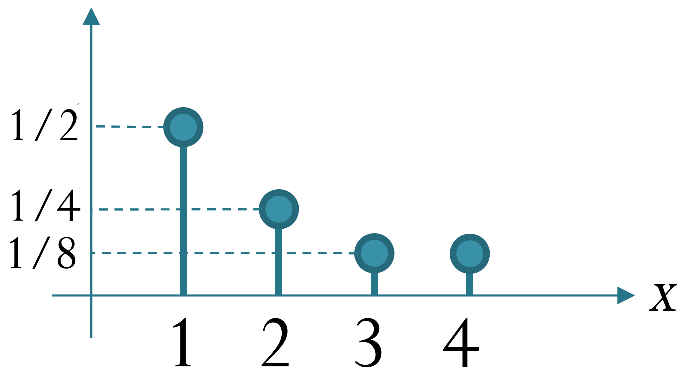


The "default" support for this RV is the set $S_X = \{1,2,3,4\}$.

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$$p_X(x) = \mathrm{P}[X = x]$$

cumulative distribution function (cdf)

$$F_X(x) = \mathrm{P}[X \le x]$$

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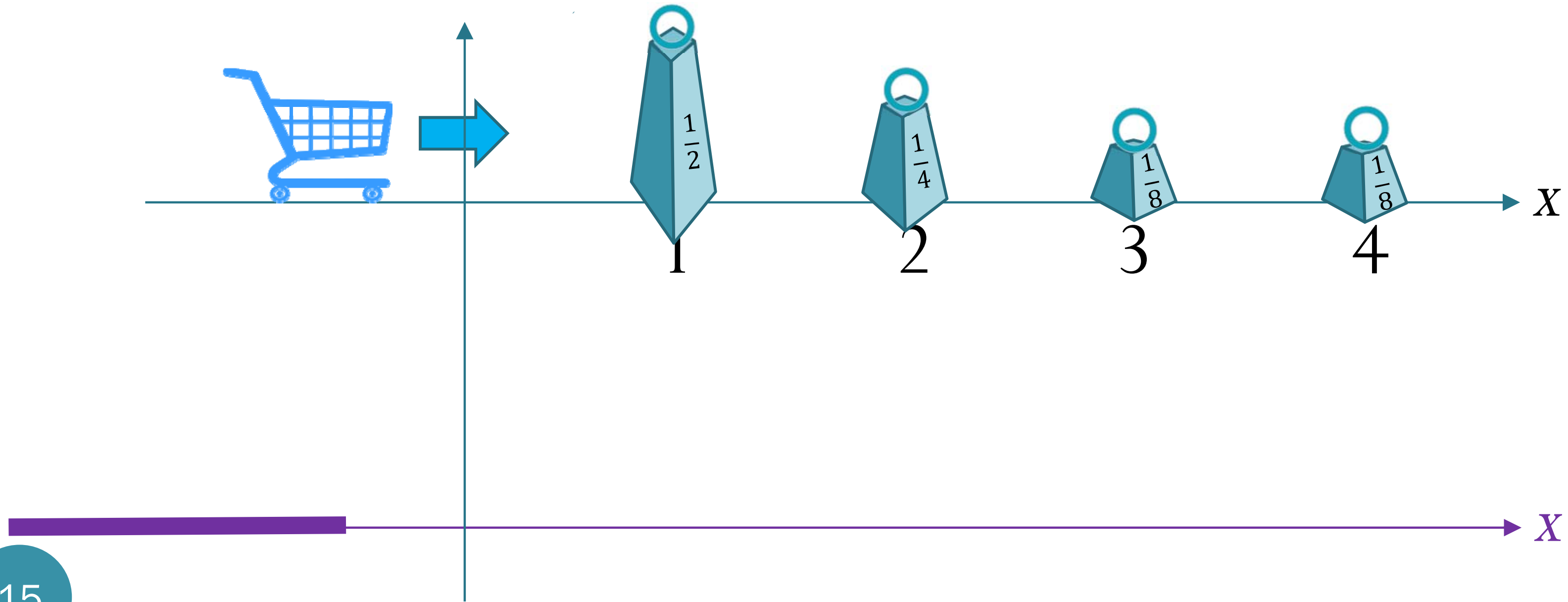

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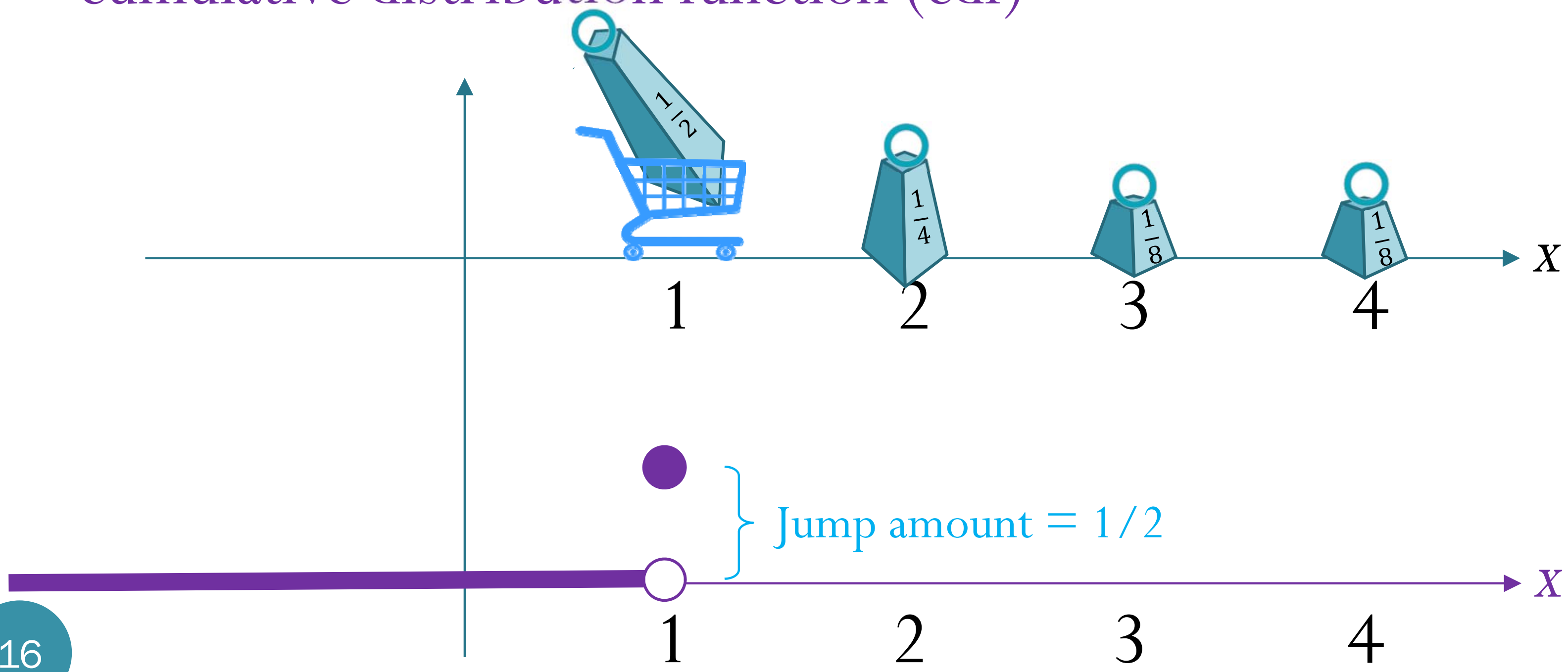

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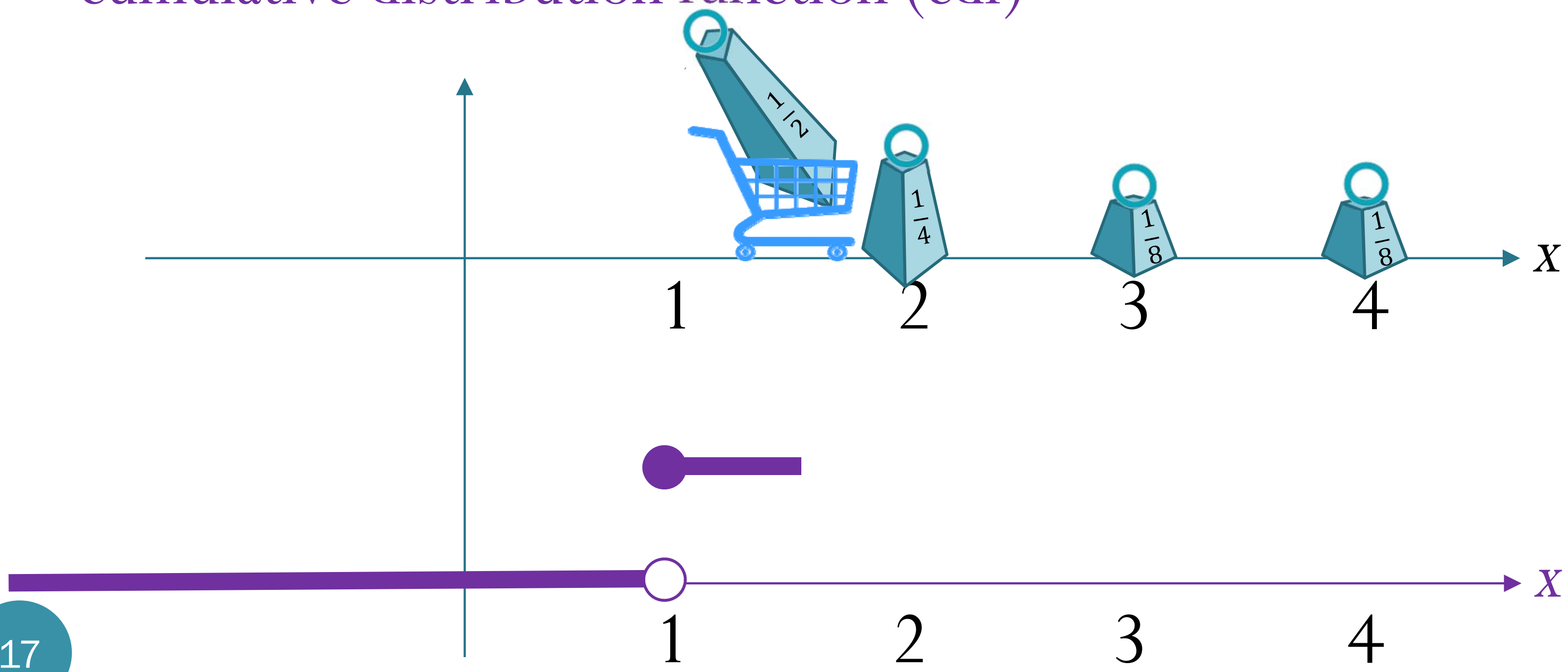

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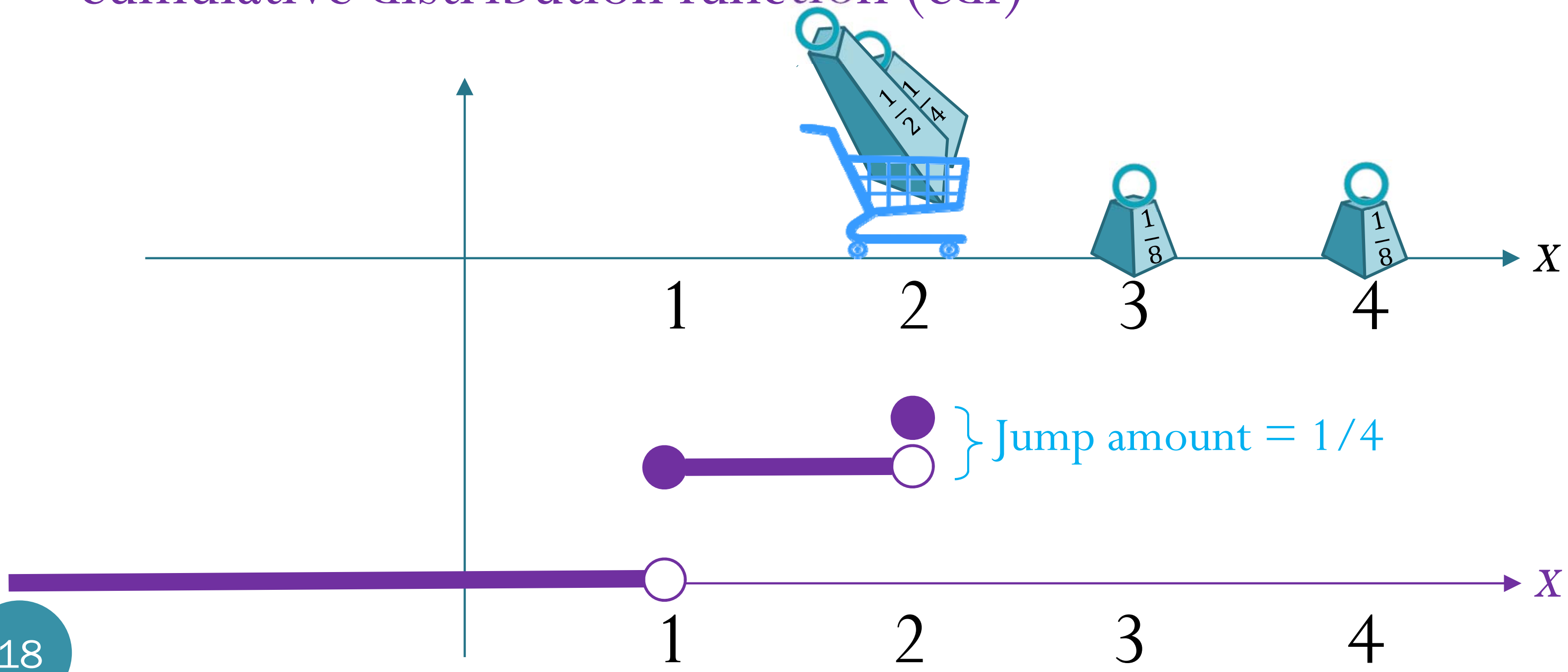

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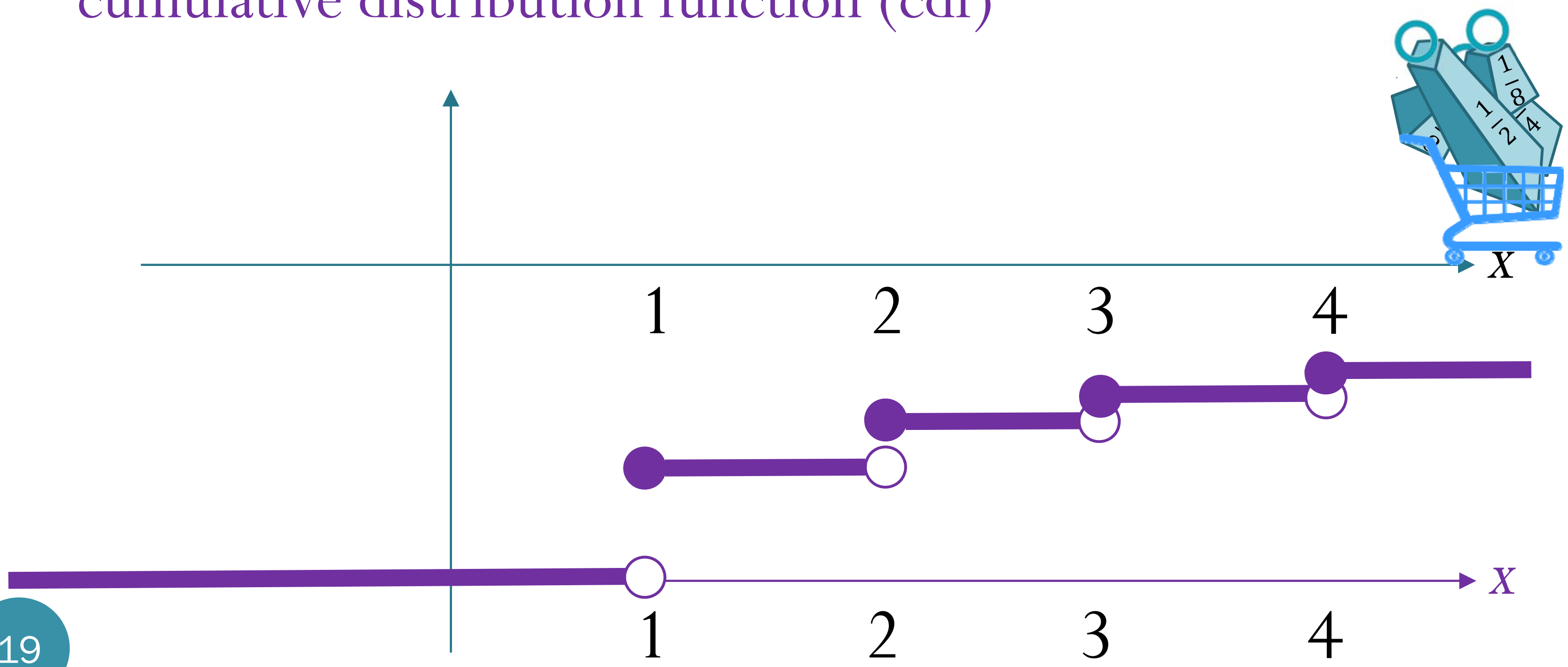

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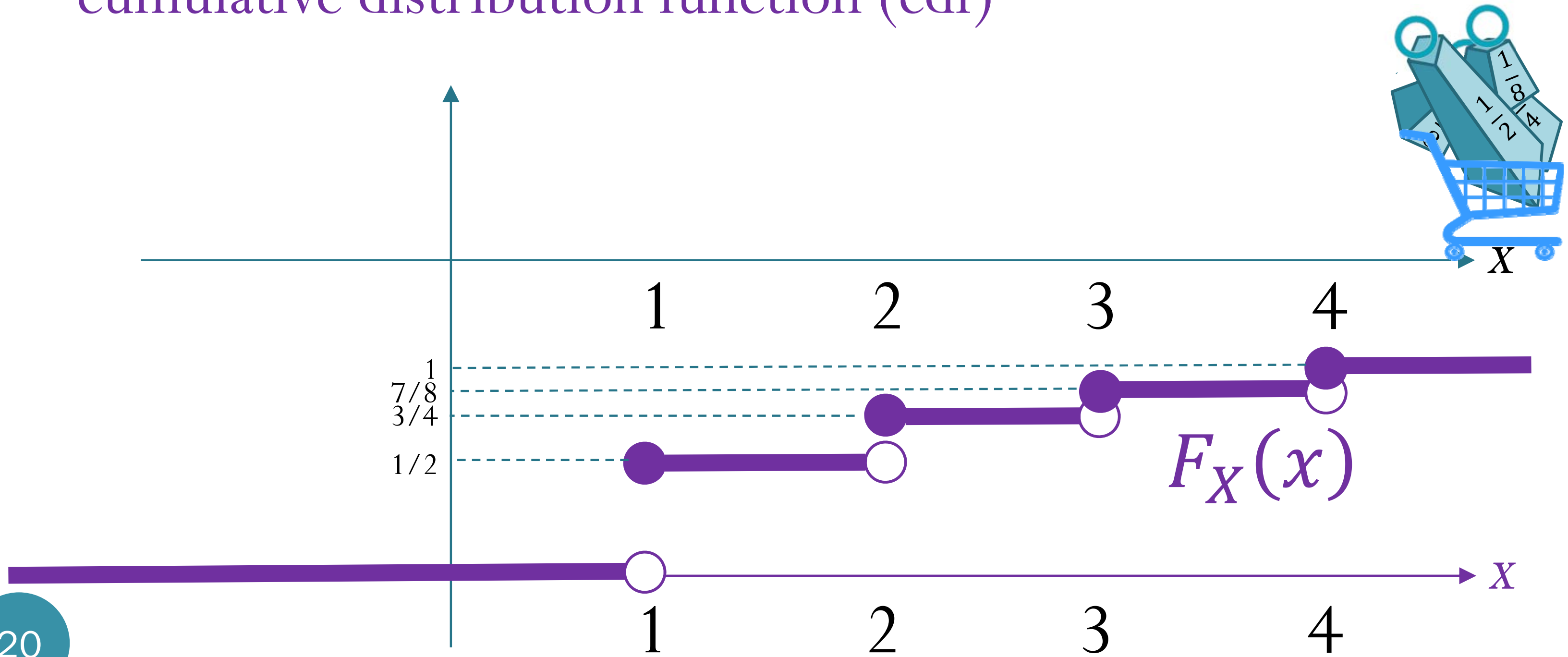

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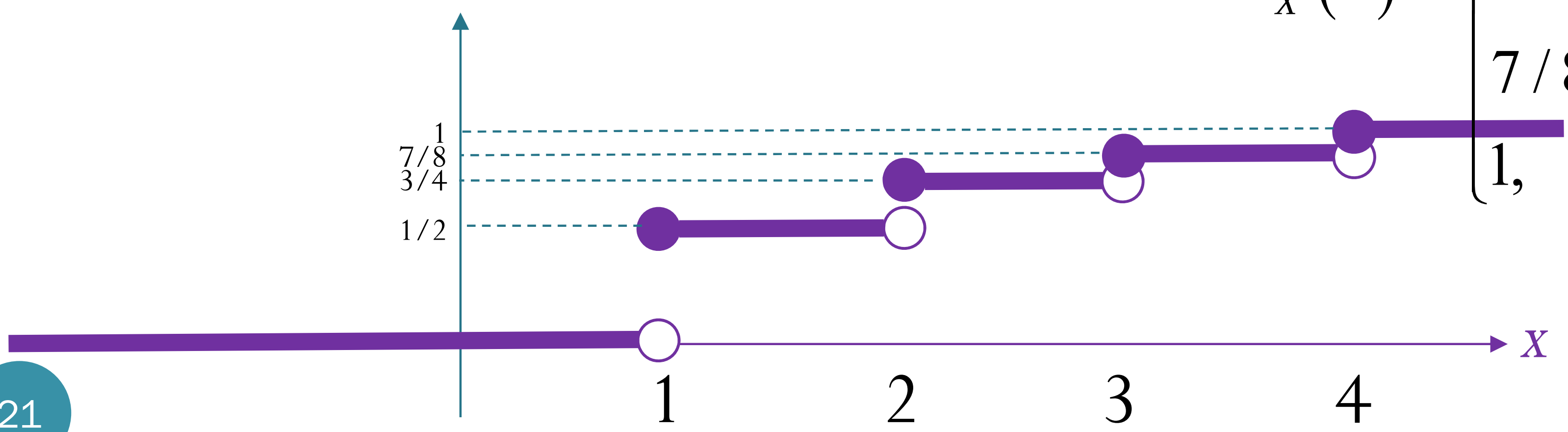
$$F_X(x) = \begin{cases} 0, & x < 1, \\ 1/2, & 1 \le x < 2, \\ 3/4, & 2 \le x < 3, \\ 7/8, & 3 \le x < 4, \\ 1, & x \ge 4. \end{cases}$$



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